A MATHEMATICAL LOGIC WITHOUT VARIABLES. II

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In the first part[1] we set up a formal system and proved that it was not too strong in the sense that it would not enable us to carry out certain (to us) undesirable types of proofs. We now prove that it is strong enough to do what we ask of it.

Section E

F1. P5 $\vdash EI$.

Proof.	$I = I \times I$	P5
	EI	M36

Strictly speaking, we should put

F1 $\rightarrow EI$
P5 $\vdash$ F1.

However, it is more convenient to write it as we have, thus stating on one line the fact that the formula which we are going to call F1 can be proved from P5. We shall in general omit explicit mention of M36, M37, or MC in the steps of a proof.

F2. P2 $\vdash EJ$.
F3. P1 $\vdash E\Pi$.
F4. P1 $\vdash E\Sigma$.
F5. P1 $\vdash ET$.
F6. P1 $\vdash EB$.
F7. P4 $\vdash EC$.
F8. P8 $\vdash EW$.
M38. $B(\mathbf{p} \times \mathbf{q})$ conv $B\mathbf{p} \times B\mathbf{q}$.

Proof.	$B(\mathbf{p} \times \mathbf{q})$ conv $(BB \times B)\mathbf{pq}$	r-conv.
	conv $(CB^3B)\mathbf{pq}$	P6
	conv $B\mathbf{p} \times B\mathbf{q}$	r-conv.

Note. $B(\mathbf{p} \times \mathbf{q})$ can be reduced to a form where no further reductions are possible. Therefore, by T14, if we perform all possible reductions on $(BB \times B)\mathbf{pq}$ we shall get the same result. By this means the reader can verify that $B(\mathbf{p} \times \mathbf{q})$ conv $(BB \times B)\mathbf{pq}$. Whenever this type of verification is possible, we omit the details of the proof. In the next step we indicate by the P6 to the right that P6 is the conversion postulate used in that step.

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