

ON CRITERIA FOR FOURIER CONSTANTS OF L -INTEGRABLE FUNCTIONS OF SEVERAL VARIABLES

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1. **Introduction.** In a note published in the Proceedings of the National Academy of Sciences I have given for Fourier constants of a function of one variable a criterion which includes several previously known criteria.¹ In a paper presented to the American Mathematical Society in December 1933 I obtained an analogous criterion for Fourier coefficients of functions of two variables.²

The criterion in the case of functions of one variable was also obtained independently by Cesari,³ who more recently has formulated for the case of functions of one variable a more general criterion which includes the previous one and a variety of other interesting special cases.⁴ It is the purpose of the present paper to obtain an analogous criterion for functions of several variables. For the sake of simplicity we shall give the detailed discussion only for the case of two variables, as the extension to the more general case is fairly obvious.

2. **Terminology.** We are given two doubly infinite sets of constants A_{mn} , C_{mn} ($m, n = 0, 1, 2, \dots$), formally related in the following manner

$$(1) \quad \sum_{m=0, n=0}^{\infty, \infty} A_{mn} x^m y^n \sim 1 / \left[\sum_{m=0, n=0}^{\infty, \infty} C_{mn} x^m y^n \right],$$

where the symbol \sim indicates that like powers of x and y on the two sides of the relationship are to be set equal to each other. For ranges of values of x and y for which the two power series converge, the relationship becomes an equality.

Let us set

$$(2) \quad S_{mn}(x, y, p, q) = \sum_{i=0, j=0}^{i=p, j=q} C_{m-i, n-j} \beta_{ij}(x, y) \quad (0 \leq p \leq m, 0 \leq q \leq n),$$

where

$$(3) \quad \begin{aligned} \beta_{ij}(x, y) &= \cos ix \cos jy \quad (i, j = 1, 2, \dots), & \beta_{00} &= \tfrac{1}{4}, \\ \beta_{0j}(x, y) &= \tfrac{1}{2} \cos jy \quad (j = 1, 2, \dots), & \beta_{i0}(x, y) &= \tfrac{1}{2} \cos ix \quad (i = 1, 2, \dots). \end{aligned}$$

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¹ Vol. 19(1933), pp. 846-848. Cf. also Bull. Amer. Math. Soc., vol. 39(1933), pp. 907-913.

² For abstract see Bull. Amer. Math. Soc., vol. 40(1934), p. 40.

³ Annali d. R. Sc. di Pisa, (2), vol. 3(1934), pp. 105-134; see, in particular, pp. 119-129.

⁴ L. Cesari, *Sulle condizioni sufficienti per le successioni di Fourier*, Boll. della Unione Mat. Ital., vol. 13(1934), pp. 100-104.