THE NEIGHBORHOOD OF A SEXTACTIC POINT ON A PLANE CURVE

By Ernest P. Lane

1. **Introduction.** In recent years projective differential geometers have manifested increasing interest in the neighborhoods of singular elements. For example, in studying the neighborhood of an ordinary point on an analytic plane curve, inflexion points are usually excluded from consideration as singular. However, Bompiani has constructed a theory[1] of the neighborhood of an inflexion point on a plane curve, which has found fruitful applications in some work[2] of Su, and also in a recent paper[3] of the author.

An inflexion point on a plane curve being defined as usual to be a point where the curve possesses a unique tangent having precisely three-point contact, the singularity which naturally presents itself next for consideration is *the sextactic point*, which is defined to be a point where the curve possesses a unique tangent having precisely two-point contact, and also possesses a proper osculating conic having precisely six-point contact with the curve. So at a sextactic point the osculating conic hyperosculates the curve in the same sense as does the inflexional tangent at an inflexion point.

In this note a brief study is made of the neighborhood of a sextactic point. In §2 a canonical power series expansion is deduced which represents an analytic plane curve in the neighborhood of a sextactic point on it. In §3 are found some applications of this expansion.

2. **Canonical power series expansion.** Let us establish a projective coördinate system in a plane, in which a point has non-homogeneous coördinates x, y and homogeneous coördinates x_1, x_2, x_3, connected by the relations $x = x_2/x_1$, $y = x_3/x_1$. The context will show in any instance which coördinates are being used. Then let us consider a curve C which, in the neighborhood of a point $O(b, c)$ on it, can be represented by a power series expansion of the form

$$y - c = a_1(x - b) + a_2(x - b)^2 + a_3(x - b)^3 + \cdots . \tag{1}$$

By suitable choice of the coördinate system this expansion can be very much simplified, and it is the purpose of this section to carry this simplification as far

Received June 6, 1935.

[1] E. Bompiani, *Per lo studio proiettivo-differenziale delle singularità*, Bollettino dell' Unione Matematica Italiana, vol. 5 (1926), p. 118.

[2] B. Su, *On certain quadratic cones projectively connected with a space curve and a surface*, Tôhoku Mathematical Journal, vol. 38 (1933), p. 233.

[3] E. P. Lane, *Plane sections through an asymptotic tangent of a surface*, Bulletin of the American Mathematical Society, vol. 41 (1935), p. 285.