CONCERNING CERTAIN REDUCIBLE POLYNOMIALS

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1. **Introduction.** In a joint paper[1] by Oystein Ore and the author, it has been shown that polynomials with rational integral coefficients which take the values $\pm p$ (p a rational prime) for m integral values of the argument must take the same value $+p$ or $-p$ for $m > 5$, and consequently have the form

$$(x - a_1)(x - a_2) \cdots (x - a_m)\, h(x) \pm p.$$

In the same paper it has also been shown that integral polynomials of the form

$$f(x) = a(x - a_1) \cdots (x - a_n) \pm p \tag{1}$$

for $n > 6$ are irreducible in the rational domain if n is odd, and if n is even they may have only two factors of the degree $n/2$.[2] A new proof of this result is contained in a recent paper by A. Brauer.[3] In this paper, Brauer also raises the question whether or not these reducible polynomials can exist for every even n. A numerical example of eighth degree for the prime 2879 has been given by Pólya,[4] and one of tenth degree for the prime 10079 is contained in Brauer's paper. However, neither of these writers has obtained any general results on the subject.

We shall here find an expression for the necessary and sufficient conditions for the reducibility of these polynomials. From this result follows first the theorem of Brauer that the sum or difference of two such factors is a constant. Furthermore, it reduces the problem to well known problems in Diophantine equations for which partial solutions exist, and these solutions in turn give

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[1] H. L. Dorwart and Oystein Ore, *Criteria for the irreducibility of polynomials*, Annals of Math., (2), vol. 34 (1933), pp. 81–94.

[2] That this theorem does not hold for $n = 6$ is shown by the following class of polynomials

$$x(x+1)(x-1)(x-2)(x-\alpha)(x-\beta) - p = [x^2 - x - 1]\,[x^4 - 2x^3 - (p+1)x^2 + (p+2)\,x + p]$$

for primes of the form $\alpha^2 - (\alpha + 1)$, where α is a positive integer > 2 and $\beta = 1 - \alpha$.

[3] A. Brauer, *Bemerkungen zu einem Satze von Herrn G. Pólya*, Jahresber. Deutschen Math. Ver., vol. 43 (1933), pp. 124–129.

[4] Georg Pólya, *Verschiedene Bemerkungen zur Zahlentheorie*, Jahresber. Deutschen Math. Ver., vol. 28 (1919), p. 40.