# ABELIAN SUBGROUPS OF ORDER $p^m$ OF THE I-GROUPS OF THE ABELIAN GROUPS OF ORDER $p^n$ AND TYPE 1, 1, 1, $\cdots$

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The group of isomorphisms $I$ of a given group is important in the construction of new groups which contain the given group as an invariant subgroup. However, little is known about groups of isomorphisms in general, and even when the given group is of the simplest type, i.e., abelian, the group of isomorphisms has not been thoroughly studied except in a few extremely special cases.

For instance, the $I$-group of the cyclic group has been discussed by Burnside[1] and Miller.[2] The $I$-group of a group of order $p^n$ and type $n - 1, 1$ was studied by Miller[3] and although the general type has been barely touched upon he has proved several general theorems.[4] The $I$-group of the abelian group $H$ of order $p^n$ and type $1, 1, 1, \cdots$ was considered by Moore,[5] and it is well known that the operators $U$ of this group can be represented as non-singular linear transformations on the exponents of $n$ independent generators of $H$. Thus a determination of the subgroups of the group of these $n$-ary linear homogeneous transformations modulo $p$ is equivalent to a determination of the subgroups of the $I$-group of $H$. Dickson determined all the subgroups in the case $n = 3$,[6] and the subgroups of order a multiple of $p$ in the case $n = 4$,[7] and all the subgroups of the three highest powers of $p$ for all positive integral values of $n$.[8]

A necessary and sufficient condition that an operator $U$ in $I$ be of order a power of $p$ is that the characteristic determinant of $U$ be $(-1)^n(\lambda - 1)^n$.[9] The invariant factors of such operators are powers of $(1 - \lambda)$ which determine the canonical form and conjugates of $U$. There is a one-to-one correspondence between these powers of $(1 - \lambda)$, the conjugate sets of operators in $I$ whose orders are powers of $p$, and the partitions of $n$. Hence, we shall designate an operator $U$ of $I$ of order $p^m$ by the degrees of its invariant factors or by the partition of $n$ to which it corresponds, i.e.,

$$n = n_1 + n_2 + n_3 + \cdots + n_\gamma + n_{\gamma+1} + \cdots + n_\delta,$$

where the terms are ordered so that $n_i \geqq n_{i+1}$, $n_\gamma > 1$, $n_{\gamma+i} = 1$, if $\gamma < \delta$.

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[1] W. Burnside, *Theory of Groups of a Finite Order*, 1897, pp. 239–242.

[2] G. A. Miller, Transactions of the Amer. Math. Soc., vol. 4 (1903), pp. 152–160.

[3] G. A. Miller, Transactions of the Amer. Math. Soc., vol. 2 (1901), pp. 259–264.

[4] G. A. Miller, Annals of Mathematics, (2), vol. 3 (1902), pp. 183–184; Amer. Journ. of Math., vol. 36 (1914), pp. 47–52.

[5] E. H. Moore, Bulletin of the Amer. Math. Soc., vol. 2 (1895), pp. 33–43.

[6] L. E. Dickson, Amer. Journ. of Math., vol. 27 (1905), pp. 189–202.

[7] L. E. Dickson, Amer. Journ. of Math., vol. 28 (1906), pp. 1–16.

[8] L. E. Dickson, Quarterly Journ. of Math., vol. 36 (1904–05).

[9] H. R. Brahana, Proc. of the Nat. Acad. of Sci., vol. 18 (1932), p. 724.