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EXOTIC PROJECTIVE STRUCTURES AND QUASI-FUCHSIAN SPACE

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1. Introduction. Let S be an oriented closed surface of genus $g > 1$. A projective structure on S is a maximal system of local coordinates modeled on the Riemann sphere $\widehat{\mathbf{C}}$, whose transition functions are Möbius transformations. For a given projective structure on S, we have a pair (f, ρ) of a local homeomorphism f from the universal cover $\widetilde{S}$ of S to $\widehat{\mathbf{C}}$, called a developing map, and a group homomorphism ρ of $\pi_1(S)$ into $\mathrm{PSL}_2(\mathbf{C})$, called a holonomy representation. Let $P(S)$ denote the space of all (marked) projective structures on S, and let $V(S)$ denote the space of all conjugacy classes of representations of $\pi_1(S)$ into $\mathrm{PSL}_2(\mathbf{C})$. Holonomy representations give a mapping $\mathrm{hol} : P(S) \to V(S)$, which is called the holonomy mapping. It is known that the map hol is a local homeomorphism (see [13]). The quasi-Fuchsian space $QF(S)$ is the subspace of $V(S)$ consisting of faithful representations whose holonomy images are quasi-Fuchsian groups.

In this paper, we investigate the subset $Q(S) = \mathrm{hol}^{-1}(QF(S))$ of $P(S)$. We say an element of $Q(S)$ is *standard* if its developing map is injective; otherwise, it is *exotic*. The set of standard projective structures with fixed underlying complex structure is well known as the image of the Teichmüller space under Bers embedding (see [5]). On the other hand, the existence of exotic projective structures was first shown by Maskit [21]. More investigations of exotic projective structures are found in [11], [12], [13], [25], [30], and [32]. As we see in Proposition 2.3, each connected component of $Q(S)$ is biholomorphically isomorphic to $QF(S)$. Moreover, as a consequence of the result of Goldman [12], the connected components of $Q(S)$ are in one-to-one correspondence with the set $\mathcal{ML}_{\mathbf{Z}}(S)$ of integral points of measured laminations. (See Section 2.4 for a precise definition.) We denote by $\mathcal{Q}_\lambda$ the component of $Q(S)$ corresponding to $\lambda \in \mathcal{ML}_{\mathbf{Z}}(S)$, where $\mathcal{Q}_0$ is the component consisting of all standard projective structures.

Recently, McMullen [25, Appendix A] discovered the following phenomenon.

THEOREM 1.1 (McMullen). *There exists a sequence of exotic projective structures that converges to a point of the relative boundary* $\partial\mathcal{Q}_0 = \overline{\mathcal{Q}_0} - \mathcal{Q}_0$ *of* $\mathcal{Q}_0$.

This phenomenon deeply depends on the following phenomenon in the theory of Kleinian groups: There is a sequence of quasi-Fuchsian groups whose algebraic limit is properly contained in the geometric limit. Such a sequence of quasi-Fuchsian

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