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INTERIOR REGULARITY OF THE COMPLEX MONGE-AMPÈRE EQUATION IN CONVEX DOMAINS

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0. Introduction. For C^2-smooth plurisubharmonic (psh) functions, we consider the complex Monge-Ampère equation

$$\det\left(u_{i\bar{j}}\right) = \psi, \tag{0.1}$$

where $u_{i\bar{j}} = \partial^2 u/\partial z_i \partial \bar{z}_j$, $i, j = 1, \ldots, n$. The main result of this paper is the following theorem.

THEOREM A. *Let Ω be a bounded, convex domain in $\mathbb{C}^n$. Assume that ψ is a C^∞ function in Ω such that $\psi > 0$ and $|D\psi^{1/n}|$ is bounded. Then there exists a C^∞-psh solution u of (0.1) in Ω with $\lim_{z\to\partial\Omega} u(z) = 0$.*

The theory of fully nonlinear elliptic operators of second order can be applied to the operator $(\det(u_{i\bar{j}}))^{1/n}$. It follows in particular that if u is strictly psh and $C^{2,\alpha}$ for some $\alpha \in (0,1)$, then $\det(u_{i\bar{j}}) \in C^{k,\beta}$ implies $u \in C^{k+2,\beta}$, where $k = 1, 2, \ldots$, and $\beta \in (0,1)$ (see, e.g., [9, Lemma 17.16]). Therefore, to prove Theorem A, it is enough to show existence of a solution that is $C^{2,\alpha}$ in every $\Omega' \Subset \Omega$, where $\alpha \in (0,1)$ depends on Ω'. We obtain this assuming only that $\psi^{1/n}$ is positive and Lipschitz in Ω (see Theorem 4.1).

In a special case of a polydisc, we also allow nonzero boundary values.

THEOREM B. *Let P be a polydisc in $\mathbb{C}^n$. Assume that ψ is a C^∞ function in P such that $\psi > 0$ and $|D^2\psi^{1/n}|$ is bounded. Let f be a $C^{1,1}$ function on the boundary ∂P such that f is subharmonic on every analytic disc embedded in ∂P. Then (0.1) has a C^∞-psh solution in P such that $\lim_{\zeta\to z} u(\zeta) = f(z)$ for $z \in \partial P$.*

In Section 5, we explain what we precisely mean by saying that a function is $C^{1,1}$ on a (nonsmooth) set ∂P. In particular, all functions that are extendable to a $C^{1,1}$ function in an open neighborhood of ∂P are allowed.

Usually, the Dirichlet problem for the complex Monge-Ampère operator is considered on smooth, strictly pseudoconvex domains in $\mathbb{C}^n$. For these, the existence of (weak) continuous solutions was proved in [1], whereas smooth solutions were obtained, for example, in [5], [10], and [11]. Here, however, we do not assume any

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