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CARLEMAN INEQUALITIES AND THE HEAT OPERATOR

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1. Introduction. The unique continuation property is best understood for second-order elliptic operators. The classic paper by Carleman [8] established the strong unique continuation theorem for second-order elliptic operators that need not have analytic coefficients. The powerful technique he used, the so-called "Carleman weighted inequality," has played a central role in later developments. In the 1950s, Aronszajn [3] and Cordes [11] generalized Carleman's result to higher dimensions. In recent years, this subject attracted attention from a great number of people. Many efforts have been made to relax the smoothness hypothesis on the coefficients (see [7], [15], [31], [2], and [16]). Using this method, Jerison and Kenig [19] obtained the strong unique continuation property for operators of the form $\triangle + V$ with $V \in L^{n/2}_{\text{loc}}$, $n \geq 3$. Further improvements have been made in considering other classes of coefficients (Fefferman-Phong class and Kato class) (see [30], [9], [12], and [24]), in extending the result to operators with first derivative terms and variable coefficients (see [32], [33], [39], and [40]), and so on (see [18], [13], [14], [23], [5], and [22] and the references therein).

For second-order linear parabolic operators with *time-independent* coefficients, the strong unique continuation property was reduced in [25] to the previously established elliptic counterparts. In particular, it is shown in [25] that if u is a solution of

$$\triangle u + \partial_t u + V(x)u = 0 \quad \text{in } S_T = \Omega \times (0,T), \ V \in L^{(n+1)/2}(\Omega),$$

$(x_0, t_0) \in S_T$, and

$$\int_{B_r(x_0)} u^2(x,t_0)\,dx \leq C_N r^N$$

for any integer N. Then $u(x,t_0) \equiv 0$ for $x \in \Omega$, and in addition, assuming that $u = 0$ on $\partial\Omega \times (0,T)$, then $u \equiv 0$ in S_T.

The reduction from time-independent parabolic equations to elliptic equations, a basic technique used in [25], relies on a representation formula for solutions of parabolic equations in terms of eigenfunctions of the corresponding elliptic operator, and therefore cannot be applied to more general equations with time-dependent coefficients (for the weak unique continuation, see also [17], [41]).

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