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ON THE SET-THEORETICAL YANG-BAXTER EQUATION

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1. Introduction. Let V be a vector space. Let $R : V \otimes V \to V \otimes V$ be an invertible linear transformation. The Yang-Baxter equation is the equality

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12} \tag{1}$$

of linear transformations on $V \otimes V \otimes V$.

Denote $\tau(w \otimes v) = v \otimes w : V \otimes V \to V \otimes V$ and $\sigma = \tau \circ R$. Then (1) is equivalent to the braid relation

$$\sigma_{12}\sigma_{23}\sigma_{12} = \sigma_{23}\sigma_{12}\sigma_{23}. \tag{2}$$

Because of this, a solution of (1) gives rise to a linear representation of the braid group B_n on $V^{\otimes n}$ for every n.

In [D], Drinfel'd raised the question of finding set-theoretical solutions of the Yang-Baxter equation. Specifically, we consider a set S and an invertible map $R : S \times S \to S \times S$. We think of the Yang-Baxter equation (1) as an equality of maps from $S \times S \times S$ to $S \times S \times S$. As in the linear case, a solution of (1) on a set S gives rise to an action of B_n on the set S^n.

By studying Poisson groups, Weinstein and Xu [WX] found a way of constructing set-theoretical solutions of the Yang-Baxter equation. Later on, Etingof, Schedler, and Soloviev [ESS] gave a complete classification of the nondegenerate set-theoretical solutions R of the Yang-Baxter equation satisfying $(\tau \circ R)^2 = \mathrm{id}$ (where $\tau(w, v) = (v, w)$).

In this paper, we present the following construction of set-theoretical solutions of the Yang-Baxter equation.

THEOREM 1. *Let G be a group. Let ξ and η be left and right actions of G on itself, denoted by $(u, v) \mapsto {}^{\xi(u)}v$ and $(u, v) \mapsto u^{\eta(v)}$, respectively. If the two actions satisfy the compatibility condition*

$$uv = \left({}^{\xi(u)}v\right)\left(u^{\eta(v)}\right), \tag{3}$$

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