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SYMPLECTIC-ORTHOGONAL THETA LIFTS OF GENERIC DISCRETE SERIES

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0. Introduction. Let F be a non-Archimedean local field of characteristic zero. In this paper we study a correspondence between representations of symplectic groups $\mathrm{Sp}(n, F)$ and special even-orthogonal split groups $\mathrm{SO}(2r, F)$, where $r \geq 2$. Let $\omega_{n,r}$ be the Weil representation of $\mathrm{Sp}(2nr, F)$ attached to a nontrivial additive character ψ_F of F. We show that the correspondence arising by restricting the Weil representation $\omega_{n,r}$ to $\mathrm{Sp}(n, F) \times \mathrm{SO}(2r, F)$ is functorial for generic square integrable representations.

More precisely, let $\mathcal{T}$ be a smooth, irreducible representation of $\mathrm{Sp}(n, F)$. Let $\Theta(\mathcal{T}, r)$ be the maximal $\mathcal{T}$-isotypic quotient of $\omega_{n,r}$. The smallest r such that $\Theta(\mathcal{T}, r) \neq 0$ is called the first occurrence index of $\mathcal{T}$. Now assume that $\mathcal{T}$ is a ψ-generic discrete series. (See (1.1) for the definition of ψ.) Let $L(s, \mathcal{T})$ be the standard L-function attached to $\mathcal{T}$ as in [Sh1]. Then we have the following results.

If $L(0, \mathcal{T}) = \infty$, then the first occurrence index is n. Let τ' be an irreducible quotient of $\Theta(\mathcal{T}, n)$. Then τ' is a ψ'-generic discrete series representation of $\mathrm{SO}(2n, F)$, and for any discrete series representation δ of $\mathrm{GL}(m, F)$ (m arbitrary), we have

$$L(s, \delta \times \mathcal{T}) = L(s, \delta) L\big(s, \delta \times \tau'\big).$$

If $L(0, \mathcal{T}) \neq \infty$, then the first occurrence index is $n+1$. Then $\Theta(\mathcal{T}, n+1)$ has the unique irreducible ψ'-generic quotient τ'. Furthermore, τ' is a discrete series representation of $\mathrm{SO}(2n+2, F)$, and for any discrete series representation δ of $\mathrm{GL}(m, F)$ (m arbitrary), we have

$$L(s, \delta \times \tau') = L(s, \delta) L(s, \delta \times \mathcal{T}).$$

We also have analogous results for ψ'-generic discrete series of $\mathrm{SO}(2n, F)$. We refer the reader to Section 2 for precise statements.

Our results have a conjectural interpretation as follows. Consider inclusions of dual groups

$$\mathrm{SO}(2n, \mathbb{C}) \subset \mathrm{SO}(2n+1, \mathbb{C}) \subset \mathrm{SO}(2n+2, \mathbb{C}).$$

Let $W'(F)$ be the Weil-Deligne group of F. The conjectural Langlands parameter of $\mathcal{T}$ is an admissible homomorphism (see [Bo])

$$\varphi : W'(F) \longrightarrow \mathrm{SO}(2n+1, \mathbb{C}).$$

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