Erratum for “An affine version of a theorem of Nagata”

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The main result of the author's paper [1] is the following.

THEOREM 2.1

Let k be a field, and let R be an affine k-algebra. Suppose that there exists a field K with $R \subset K^{[1]}$ and $R \not\subset K$. Then there exists a field F and an algebraic extension $R \subset F^{[1]}$.

The subsequent Theorem 3.1 deals with the case where the transcendence degree of R over k is one, but without assuming that R is affine.

THEOREM 3.1

Let k be a field, and let R be a k-algebra with $\mathrm{tr.deg}_k R = 1$. Suppose that there exists a field K with $R \subset K^{[1]}$ and $R \not\subset K$. Then R is k-affine, and there exists a field F algebraic over k with $R \subset F^{[1]}$. If k is algebraically closed, then there exists $t \in \mathrm{frac}(R)$ with $R \subset k[t]$.

However, when the ground field k is not algebraically closed, the ring R may fail to be affine. For example, let K/k be an extension of fields that is not a finite extension. Define $W \subset K[s] = K^{[1]}$ by $W = \{as \mid a \in K\}$, and define $R = k[W]$. Then $R \subset K[s]$ is an algebraic extension and $R \not\subset K$, but R is not affine over k.

The purpose of this note is to give the correct version of Theorem 3.1, as follows.

THEOREM 3.1 (CORRECTED)

Let k be a field, and let R be a k-algebra with $\mathrm{tr.deg}_k R = 1$. Suppose that there exists a field K with $R \subset K^{[1]}$ and $R \not\subset K$. Then there exists a field F algebraic over k with $R \subset F^{[1]}$. If k is algebraically closed, then R is k-affine, and there exists $t \in \mathrm{frac}(R)$ with $R \subset k[t]$.

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