ERRATA TO "SOLUTIONS OF SUPER-LINEAR ELLIPTIC EQUATIONS AND THEIR MORSE INDICES, I," DUKE MATH. J. 94 (1998), 141–157

SALEM REBHI

We first point out that the second estimate in Lemma 2.6 (p. 150) is false. Indeed, let $G_{B_1^+}(x,y)$ be the Green function of $B_1^+ = \{x \in \mathbb{R}^N, |x| < 1, x_N > 0\}$. We then have

$$-\Delta_y G_{B_1^+}(x,y) = \delta_y(x), \quad \text{in } B_1^+ \qquad \text{and} \qquad G(x,y) = 0 \quad \text{if } y \in \partial B_1^+.$$

Hence

$$\int_{\partial B_1^+} \frac{\partial G_{B_1^+}(x,y)}{\partial n_y}\, d\sigma = \int_{B_1^+} \Delta G_{B_1^+}(x,y)\, dy = -1, \quad \forall x \in B_1^+,$$

which implies that

$$\int_{\partial B_1^+} \frac{\partial^2 G_{B_1^+}(x,y)}{\partial x_N\, \partial n_y}\, d\sigma = 0, \quad \forall x \in B_1^+.$$

Then we cannot have

$$\frac{\partial^2 G_{B_1^+}(x,y)}{\partial x_N\, \partial n_y} > 0 \quad \text{on } \partial B_1^+ = S_1^+ \cup (\pi \cap B_1)$$

wherever the point $x \in B_1^+$ is.

However, the first estimate of [1, Lemma 2.6] on $S_1^+ = \{x \in \partial B_1^+, x_N > 0\}$ holds true. That is, when $x = (x', x_N)$, there exists $\theta > 0$ and $c > 0$ such that if $|x'| \le 1/2, 0 < x_N < \theta$, we have

$$0 < \frac{\partial^2 G_{B_1^+}(x,y)}{\partial x_N\, \partial n_y} \le c, \quad \forall y \in S_1^+. \tag{0.1}$$

Since in the proof of [1, Lemma 2.7] we used the second estimate of Lemma 2.6 (see line 5, p.153), we indicate how to overcome this problem.

The function $w = 1 + v - u$ satisfies $-\Delta w = 0$ in $\mathbb{R}^N_+ w \ge 0$. For $R > 0$ and for all $x \in B_R^+$, with $0 < x_N < R\theta$ and $0 < |x'| < \frac{R}{2}$, we have

DUKE MATHEMATICAL JOURNAL
Vol. 162, No. 1, © 2013 DOI 10.1215/00127094-1959328
Received 23 July 2012.
2010 *Mathematics Subject Classification*. Primary 35J60.