J. Math. Kyoto Univ. (JMKYAZ)
49-4 (2009), 691–717

Decaying solution of a Navier-Stokes flow without surface tension

By

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1. Introduction

We consider an initial boundary value problem for the motion of a Navier-Stokes flow in a 3D domain with free boundary. Under the periodic boundary condition in horizontal directions, we discuss the global existence of small smooth solution with decaying property in time. The surface tension effect is not taken into account throughout this paper.

The domain which the fluid occupies is bounded below by a rigid flat floor and above by an atmosphere of constant pressure; the upper free surface moves with the change of motion of the fluid. We introduce a spatial coordinate system $\boldsymbol{x}' = (x_1, x_2)$ and $\boldsymbol{x} = (\boldsymbol{x}', x_3)$, and assume that the domain of fluid at time t is described by $\Omega(t) = \{\boldsymbol{x} \in \mathbb{T}^2 \times \mathbb{R} : -1 < x_3 < \eta(t, \boldsymbol{x}')\}$, which is bounded below by a fixed bottom $S_B = \{(\boldsymbol{x}', -1) : \boldsymbol{x}' \in \mathbb{T}^2\}$, and above by a free surface $\Gamma(t) = \{(\boldsymbol{x}', \eta(t, \boldsymbol{x}')) : \boldsymbol{x}' \in \mathbb{T}^2\}$. We denote the velocity of fluid by $\boldsymbol{v} = {}^t(v_1, v_2, v_3)$, and by p a correction to the hydrostatic pressure $\bar{p}$ as $p = \bar{p}(t, \boldsymbol{x}) - P_0 + \rho g x_3$, where P_0 is the atmospheric pressure above the fluid, ρ is a constant density and g is the acceleration of gravity. The equations describing the motion of fluid are given as follows:

$$\boldsymbol{v}_t + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} + \nabla p - \nu \Delta \boldsymbol{v} = \boldsymbol{0} \quad \text{in } \Omega(t) \tag{1.1}$$

$$\nabla \cdot \boldsymbol{v} = 0 \quad \text{in } \Omega(t) \tag{1.2}$$

$$\eta_t + \boldsymbol{v} \cdot \nabla' \eta - v_3 = 0 \quad \text{on } \Gamma(t) \tag{1.3}$$

$$p n_j - \sum_{k=1}^{3} \nu (v_{j,x_k} + v_{k,x_j}) n_k = g\eta\, n_j \quad \text{on } \Gamma(t),\ j = 1, 2, 3 \tag{1.4}$$

$$\boldsymbol{v} = \boldsymbol{0} \quad \text{on } S_B. \tag{1.5}$$

Here ν denotes a positive viscous constant, and (n_1, n_2, n_3) the outward normal to the free surface. We denote spatial derivatives by $\nabla' = {}^t(\partial/\partial x_1, \partial/\partial x_2, 0)$, $\nabla = {}^t(\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$, and $\Delta = \nabla \cdot \nabla$. Subscripts after comma denote derivatives, and bold type letters mean vectors. The equilibrium state to this

Received May 20, 2004
Revised April 6, 2009