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The Theorem of BERTINI on Linear Systems in Modular Fields.

By

Teruhisa Matsusaka

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O. Zariski[1] has clarified algebraically the proof of the so-called theorem of Bertini concerning the reducible linear systems on algebraic varieties in the projective space over the ground field of characteristic zero. We wish to investigate it over general ground field of any characteristic.

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Following A. Weil[2], we fix once for all the universal domain Ω, of the given characteristic p. We understand under the "extension of a field k" the subfield of Ω which is finitely generated over k by a set of quantities.

§ I Preliminaries (I)

Let a field Σ be an extension of a field k; the derivations of Σ over k form a Σ-module and we denote it by $\mathfrak{D}(\Sigma/k)$. Since Σ is an extension of k, the rank of $\mathfrak{D}(\Sigma/k)$ with respect to Σ is finite.

Let V^r be a Subvariety of a projective n-space (F-Appendix I) and assume that it is everywhere relatively normal with reference to some field of definition k for V. (F-Appendix II). Let V_α be a representative of V and M or M_α be a generic Point of V or V_α respectively over k. Then it holds $\Sigma = k(M) = k(M_\alpha)$ and the ring $k[M_\alpha]$ is integrally closed in Σ. It is well-known that the (r−1)-dimensional irreducible Subvariety of V over k (prime

1) O. Zariski. Pencils on an algebraic variety and a new proof of a theorem of Bertini. Trans. Amer. Math. Soc. 1941. We indicate this by (Z).

2) A. Weil. Foundations of algebraic geometry. We indicate this by F-..