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New Treatise of Solutions of a System of Ordinary Differential Equations and its Application to the Uniqueness Theorems.

By

Kyuzo HAYASHI and Taro YOSHIZAWA

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§ 1. Introduction.

To explain simply let us consider the differential equation $\frac{dy}{dx}=f(x, y)$ defined in a domain of two dimensions. To each point (x, y) of the domain we adjoin the direction $f(x, y)$; so we have a manifoldness of points and directions. To solve the differential equation is nothing but to find a locus of point and direction in the manifoldness such that, for all (x, y), $f(x, y)$ shall be equal to $\frac{dy}{dx}$. Therefore we consider a curve $y=\varphi(x)$ say in the domain; then its (x, y) and $\varphi'(x)$ construct a locus, generally different from a locus in the manifoldness. Then at each point on the curve $y=\varphi(x)$, the deviation of its direction $\varphi'(x)$ from that of the manifoldness is given by $|\varphi'(x)-f(x, \varphi(x))|$. Along the curve $y=\varphi(x)$ we sum up the deviations, i. e., the totaldeviation is $\int|\varphi'(x)-f(x, \varphi(x))|\, dx$. Such quantity is considered in statistics. If it be zero, $y=\varphi(x)$ is possibly a solution of the differential equation. Thus we arrive at the idea given below (2). We remark that naturally we may define many other measures of deviations.

In this paper we shall perform integrating operations always in the sense of Lebesgue.

Consider a system of ordinary differential equations