Memoirs of the College of Science University of Kyoto, Series A,
Vol. XXVI, Mathematics No. 3, 1951.

On Generalized Spaces which admit given Holonomy Groups.

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(Received January 25, 1951)

Introduction

The properties of generalized spaces (in the sense of E. Cartan) admitting various holonomy groups have been investigated mainly in concrete examples by many authors. The present purpose is to establish, by applying the Cartan's theory of continuous groups, a general theory which includes their results as its special cases or which may avail to treat systematically concrete cases.

In order to attain this, it will be necessary to extend the *fundamental theorem* of holonomy groups which demands us to take a very restricted *moving frame of reference* (repère mobile).

The first section will be concerned with the equations which are to be satisfied by Pfaffian forms of connection in the extended fundamental theorem. In the second section we shall mainly deal with the structure of the space with general connection whose holonomy group is intransitive or imprimitive.

I. Ricci's Relations of a subgroup.

1. Ricci's families. Let $\mathfrak{G}=\{T_a\}$ be a transitive Lie group of transformations which have been defined on a Klein space $\boldsymbol{E}^n$ of points ξ^i. Let $a^1,\dots,a^r$ be the parameters of $\mathfrak{G}$ and $T_0=T_{a=0}$ the identical transformation.

Let R_a be a moving frame of $\mathfrak{G}$ and $\omega^1(a,da),\dots,\omega^r(a,da)$ its relative components: the symbol of an infinitesimal transformation $T_a^{-1}T_{a+da}$ can be written as

$$\omega^s(a,da)X_s,$$

where $(X_1,\dots,X_r)$ is a fixed set of r independent operators of $\mathfrak{G}$ taken so that $X_{n+1},\dots,X_r$ may generate the subgroup which fixes