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On the Varieties of the Classical Groups in the Field of Arbitrary Characteristic

by

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Let $\boldsymbol{K}$ be a universal domain over the prime field ρ of characteristic p, then the matrix of degree $n(n\geq 1)$ with coefficients in $\boldsymbol{K}$

$$\sigma=(x_i^{(j)})(1\leq i,\ j\leq n)$$

can be considered as a point of an n^2-space in our algebraic geometry. Since the equation

$$\det\ (\sigma)=0$$

in the n^2 coefficients of σ is absolutely irreducible, it defines over ρ a variety of n^2-1 dimension in the n^2-space. If we take out this variety as a frontier from the space, the abstract variety so obtained forms the *general linear group* $GL(n, \boldsymbol{K})$ by matrix multiplication. Moreover since the group operation

$$(\sigma,\ \tau)\rightarrow\sigma\cdot\tau^{-1}$$

in $GL(n, \boldsymbol{K})$ is a function, which is everywhere defined on the product variety $GL(n, \boldsymbol{K})\times GL(n, \boldsymbol{K})$, the group $GL(n, \boldsymbol{K})$ is the so-called *group variety* in the recent terminology.[1)]

We shall now define the *special linear group* $SL(n, \boldsymbol{K})$ and the *special orthogonal group* $SO(n, \boldsymbol{K})$ in $\boldsymbol{K}$ by the equations

$$\det\ (\sigma)=1$$

and $${}^t\sigma\cdot\sigma={}^t\sigma\cdot I_n\cdot\sigma=I_n,\quad \det\ (\sigma)=1$$

respectively, where I_n means the unit matrix of degree n. More-

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We shall use freely the results and terminology of Weil's book: *Foundations of algebraic geometry*, Am. Math. Soc. Colloq., Vol. 29 (1946).

1) See A. Weil, *Variétés Abéliennes et courbes algébriques*, Act. Sc. et Ind., n° 1964 (1948), § II.