MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES, A
Vol. XXVII, Mathematics No. 2, 1952.

Some Applications of Bochner's Method to Riemannian Manifolds.

By

Makoto MATSUMOTO

The well-known treatise of S. Bochner [1, 2] is based upon Green's theorem and Ricci's identity. Let V_n be a compact, orientable Riemannian manifold, whose metric is given by the positive definite quadratic form:

$$ds^2=g_{ij}dx^i dx^j.$$

Hereafter, unless otherwise stated, we shall denote by V_n a Riemannian manifold as above mentioned.

If we put, for r-tensors $\varphi_{i_1\dots i_r}$ and $\psi_{i_1\dots i_r}$,

$$(\varphi\cdot\psi)=\varphi_{i_1\dots i_r}\psi^{i_1\dots i_r},$$

$$(\varphi'\cdot\psi')=\varphi_{i_1\dots i_r;j}\,\psi^{i_1\dots i_r;j},$$

and denote by $\Delta\varphi_{i_1\dots i_r}$ the Laplacian of $\varphi_{i_1\dots i_r}$; i. e.

$$\Delta\varphi_{i_1\dots i_r}=\varphi_{i_1\dots i_r;j;k}\,g^{jk},$$

we have clearly

$$\frac{1}{2}\cdot\Delta(\varphi\cdot\varphi)=(\Delta\varphi\cdot\varphi)+(\varphi'\cdot\varphi').$$

And Green's theorem gives that

$$\int_{V_n}\Delta(\varphi\cdot\varphi)dv=0;$$

where dv is n-dimensional volume element. The other hand, we define operator D and its dual D^* as follows:

$$D\xi_{i_1\dots i_{p+1}}=\delta^{a_1\dots a_{p+1}}_{i_1\dots i_{p+1}}\xi_{a_1\dots a_p;a_{p+1}},$$

$$D^*\xi_{i_1\dots i_{p-1}}=\xi_{i_1\dots i_{p-1}j;k}\,g^{jk}.$$

In above definitions $\xi_{i_1\dots i_p}$ is a skew-symmetric p-tensor and $\delta^{b_1\dots b_r}_{a_1\dots a_r}$