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On the Genus of Algebraic Curves.

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1. It is well known that the genus of any curve of an algebraic system is not greater than the genus of the generic curve of this system. Recently W. L. Chow has given the algebraic proof of the above theorem.[1] We shall give here the complementary result to this, i. e.

Theorem 1. *If the generic curve of an algebraic syctem has no multiple point, then any irreducble member in this system without singular points, has the genus not less than that of the generic curve of this system.*

Combinig these two results we have the following

Theorem 2. *Under the same hypothesis as in Theorem 1, any irreducible member in the algebraic system of algebraic curves has the same genus as that of the generic curve, provided the former has no singular point.*

2. Let V^r be a projective model of an algebraic variety immersed in the projective space L^N, defined over k, and such that V has no singular point. Let $\varphi_i(X)$ $(i=0,1,\dots,n)$ be the homogeneous forms of same degree in $(X)=(X_0, X_1, \dots, X_N)$, then the forms $\varphi_i(X)$ determines a linar system $\sum$ on V. Let $P=(x_0, x_1, \dots, x_N)$ be the generic point of V over k and Q be the point in L^n whose homogeneous coordinates are $(\varphi_0(x), \varphi_1(x), \dots, \varphi_n(x))$. Then the point $P\times Q$ has the locus W in $V\times L^n$. Tet W' be the proiection of W on L^n, then it can readily be seen that the variety W' is not contained in any linear subvariety of L^n if and only if the forns $\varphi_0(X)$, $\varphi_1(X),\dots,\varphi_n(X)$ are linearly independent on V. In this case we shall say that the variety W' *belongs* to the projective space L^n. Let $U^s(s<r)$ be any simple subvariety of V, defined over the field K(containing k), such that U has no singular point and the projection from W to V is regular along U.

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1) W. L. Chow, on the genus of curves of an algebraic system. Trans. Amer. Math. Soc. Vol 65 (1949), pp 137–140.