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On the Existence of Systems of Periodic Solutions for Several Nonlinear Circuits.

By

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The authour has obtained with M. Yamaguti a theorem on the existence of periodic solutions for nonlinear differential equations by a simple method.[1] Now we consider the natural extension of this method to nonlinear systems. Recently D. Graffi[2] has proved the existence of periodic systems for a type of nonlinear circuits. Here we will show a general principle which guarantees the existence of periodic systems for this type of nonlinear system, i. e.

$$\sum_{j=1}^{n} L_{ij}\ddot{x}_j + f_i(x_i)\dot{x}_i + \varphi_i(x_i) = p_i(t) \quad (i=1, 2, \ldots, n), \tag{1}$$

where $L_{ij}=$const., $L_{ij}=L_{ji}$, $\sum_{i,j} L_{ij}\xi_i\xi_j > 0$, when $|\xi_1| + \ldots |\xi_n| \neq 0$, $p_i(t) \equiv p_i(t+\omega)$, $\int_0^{\omega} p_i(t)dt = 0$, $(i=1, 2, \ldots, n)$, and the functions $f_i(x_i)$, $\varphi_i(x_i)$ are continuous and moreover the latter fulfil the condition of Lipshitz,[3] and $p_i(t)$ are continuous.

And, as examples, we will show Graffi's example (example I) and another of van der Pol's type (example II).

The principle is as follows:

THEOREM. The system (1) possesses at least one system of periodic solution $(x_1(t), \ldots, x_n(t))$, $(x_i(t+\omega) \equiv x_i(t))$, *if the following conditions are fulfilled,*

i) $\operatorname{sgn} x_i \cdot \varphi_i(x_i) > 0$ for $|x_i| > q$, $\Phi_i(x_i) = \int_0^{x_i} \varphi_i(x_i)dx_i \to +\infty$, $(|x_i| \to \infty)$, $(i=1, 2, \ldots, n)$.

ii) *there exist two constants* r_0 *and* ε *such that*

$$A(x;\, t)^{4)} = \sum_{i,j} \tilde{L}_{ij}\varphi_i(x_i)\,[F_j(x_j) - P_j(t)] \geq \varepsilon(>0)$$

for $\sqrt{x_1^2 + \ldots + x_n^2} \geq r_0$, *where* $F_i(x_i) = \int_0^{x_i} f_i(x_i)dx_i$, $P_i(t) = \int_0^t p_i(t)\,dt$