MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES, A
Vol. XXVII, Mathematics No. 2, 1951.

On the Existence of Periodic Solutions of the Non-linear Differential Equation, $\ddot{x}+a(x)\cdot\dot{x}+\varphi(x)=p(t)$

By

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(Received January 31, 1952)

Many authors have investigated the conditions for the existence of the periodic solutions of the above differential equation; Nagumo[1], Furuya[2], Cartwright and Littlewood[3], Cartwright[4], and Reuter.[5] Now we prove it under weaker conditions by a simple method.

Theorem. The given differential equation, where $p(t)$ is periodic of period ω, and $\int_0^\omega p(t)dt=0$, posseses at least one periodic solution of period ω, if the following condtions are fulfilled:

a) $A(x)=\int_0^x a(x)dx \to \pm\infty$, for $x\to\pm\infty$ resp.

b) $\operatorname{sgn} x\cdot\varphi(x)\geqq 0$, for $|x|>q$

where $a(x)$, $\varphi(x)$, $\varphi'(x)$, $p(t)$ are continuous functions and q is a positive number.

Proof. Put

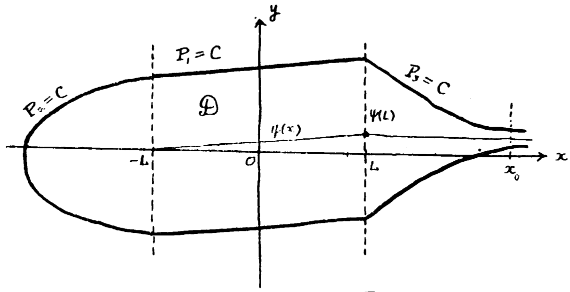

Fig. 1

$$\dot{x}=y-A(x)+\int_0^t p(t)dt=y-A(x)+P(t),\quad \dot{y}=-\varphi(x)$$