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On the integral closure of an integral domain

By

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Introduction. By an integral domain we mean a commutative ring $\mathfrak{R}$ which satisfies the following condition: $\mathfrak{R}$ satisfies the ascending chain condition and possesses no zero-divisor $\neq 0$. A local ring is a commutative ring $\mathfrak{R}$ with an unit element in which:

(1) The set $\mathfrak{p}_0$ of all non-units is an ideal in $\mathfrak{R}$;

(2) Every ideal in $\mathfrak{R}$ has a finite basis.

A local ring $\mathfrak{R}$ is called a local domain if the ring $\mathfrak{R}$ possesses no zero-divisor.

Let $\mathfrak{R}$ be an integral domain and K be the field of quotients of $\mathfrak{R}$. It is conjectured by Krull [2, p. 108] that the integral closure $\overline{\mathfrak{R}}$ of $\mathfrak{R}$ in K is an "Endliche diskrete Hauptordnung". If $\mathfrak{R}:\overline{\mathfrak{R}}\neq(0)$, $\overline{\mathfrak{R}}$ is a Noetherian ring and also Krull's conjecture is valid [2, p. 105]. Therefore it only remains that his conjecture should be proved in the case where $\mathfrak{R}:\overline{\mathfrak{R}}=(0)$. When $\mathfrak{R}$ is a 1-dimensional local domain, it was already proved by Krull [1]. Hence it is clear that Krull's conjecture is valid provided that an integral domain $\mathfrak{R}$ is "einartig" [2, p. 109]. The purpose of this paper is to prove that Krull's conjecture is valid in the case where $\mathfrak{R}:\overline{\mathfrak{R}}=(0)$ and $\mathfrak{R}$ is not "einartig".

In the first part of this paper we shall prove that Krull's conjecture is valid if the completion $\mathfrak{R}^*$ of a local domain $\mathfrak{R}$ possesses no nilpotent element. The second part is devoted to the proof of Krull's conjecture in the case in which $\mathfrak{R}^*$ has nilpotent elements, and we shall prove that Krull's conjecture is generally valid in an integral domain. In the third part we discuss the sufficient condition that $\mathfrak{R}:\overline{\mathfrak{R}}\neq(0)$ holds for a local domain.

In this paper we denote the completion of a local ring $\mathfrak{R}$ by $\mathfrak{R}^*$ and the integral closure of an integral domain $\mathfrak{S}$ in the field of quotients of $\mathfrak{S}$ by $\overline{\mathfrak{S}}$.

Numbers in brackets refer to the Bibliography at the end of the paper.