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Stochastic Differential Equations in a Differentiable Manifold (2)

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§ **1.** Let $\pi(t)$ be any Markov process in an r-dimensional differentialble manifold M with the transition probability:

$$(1\cdot1) \qquad F(t, p\,; s, E) = P(\pi(s) \in E/\pi(t) = p).$$

As is well-known, the generating operator A_t of this process is defined as follows:

$$(1\cdot2) \qquad (A_t f)(p) = \lim_{\Delta\to+0}\frac{1}{\Delta}\int_M [f(q)-f(p)]\; F(t, p;\, t+\Delta,\, dq).$$

We shall consider here the process whose generating operator A_t is expressible in the form:

$$(1\cdot3) \qquad (A_t f)(x) = a^i(t, x)\frac{\partial f}{\partial x^i}(x) + \frac{1}{2}\, B^{ij}(t, x)\frac{\partial^2 f}{\partial x^i\, \partial x^j}(x),$$

where x is the local coordinate and f is a bounded function of class C_1. (1.3) is equiualent to the following (1.3′):

$$(1.3') \begin{cases} \dfrac{1}{\Delta}\displaystyle\int_U (y^i - x^i) F(t, x;\, t+\Delta,\, dy) \longrightarrow a^i(t, x), \\ \dfrac{1}{\Delta}\displaystyle\int_U (y^i - x^i)(y^j - x^j) F(t, x\,;\, t+\Delta,\, dy) \longrightarrow B^{ij}(t, x), \;(\Delta\to+0) \\ \dfrac{1}{\Delta}\displaystyle\int_U F(t, x\,;\, t+\Delta,\; U^c) \to 0. \end{cases}$$

We can easily see that (B^{ij}) is symmetric and positive-definite and that a^i and B^{ij} are transformed in the following way:

$$(1.4') \qquad \begin{aligned} \bar{a}^i &= a^k \frac{\partial \bar{x}^i}{\partial x^k} + \frac{1}{2} B^{kl}\, \frac{\partial^2 \bar{x}^i}{\partial x^k \partial x^l} \\ \bar{B}^{ij} &= B^{kl}\frac{\partial \bar{x}^i}{\partial x^k}\, \frac{\partial \bar{x}^j}{\partial x^l} \end{aligned}$$