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Bergman kernel function and canonical slit-mapping.

By

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1. Let D be a finitely connected domain in the z-plane which contains the points $z=0$ and $z=\infty$, and bounded by n proper continua. According to the well-known Grunsky's theorem[1] in the theory of conformal mapping of multiply-connected domains there exists one and only one function which, in the neighborhood of $z=\infty$, has a Laurent expansion of the form

$$w=s_0(z)=z+\frac{c}{z}+\cdots, \tag{1}$$

and at the origin $s_0(0)=0$ and $s'_0(0)=a_0$, and which maps D conformally onto a whole plane slit along n arcs on a finite number of logarithmic spirals having the same angle of inclination $\theta/2$ and the same asymptotic point $z=0$.

In the present paper we shall derive an inequality involving the coefficient a_0 appearing in (1) and the outer logarithmic area L of the complement (with respect to the whole plane) of the domain D, namely:

$$Re(-e^{-i\theta}\log a_0)-\frac{|\log a_0|^2}{\log(A/B)}\geqq\frac{L}{2\pi}, \tag{2}$$

where A and B are constants which will be explained in the section 3.

It suffices to prove the inequality (2) in the case when the boundary continua of D are closed analytic curves $C_1, C_2\cdots,C_n$, for it is known that D can be approximated by an increasing sequence of domains having such boundaries for which the mapping functions corresponding to (1) will converges to $s_0(z)$, so that (2) will continue to hold in the limit, when L is interpreted in the manner explained above.