MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES, A
Vol. XXVIII, Mathematics No. 1, 1953.

On the Evaluation of the Derivatives of Solutions of $y''=f(x, y, y')$.

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(Received March 30, 1953)

1. Theorem. The principal part of the possibility of the prolongation of solutions of the ordinary differential equation of the second order,

$$\frac{d^2y}{dx^2}=f\left(x, y, \frac{dy}{dx}\right), \tag{1}$$

is on the evaluation of the derivatives of the solutions. On this subject Prof. Nagumo has given a sufficient condition in Proc. Physico-Math. Soc. of Japan, 3rd Series. Vol. 19 (1937)., and the late Prof. Okamura has obtained a more general and easier result in Functional Equations (in Japanese), Vol. 27, but it is also a sufficient condition.

Recently we have obtained a necessary and sufficient condition for the evaluation, by aid of the D-function having the properties like the distance has which Okamura had utilized in his research of necessary and sufficient conditions for the uniqueness of solutions in the Cauchy-problem.

Our theorem runs as follows.

Let $\mathfrak{L}$ *be a bounded closed domain in xy-plane and* $\mathfrak{L}^*$ *be a three dimensional domain of* (x, y, y'), *where* $(x, y) \in \mathfrak{L}$ *and* $-\infty < y' < +\infty$. *Let* $f(x, y, y')$ *be defined and continuous in* $\mathfrak{L}^*$. *In order that, given a positive number* α *and for a suitable positive number* $\beta(\alpha)$ $(>\alpha)$, *if for any solution* $y=y(x)$ *of* (1) *through a point* (x_0, y_0) *arbitrary in* $\mathfrak{L}$, *provided* $|y'(x_0)| \leqq \alpha$, *we have*

$$|y'(x)| < \beta(\alpha),$$

so long as $y=y(x)$ *lies in* $\mathfrak{L}$ *for* $x_0 \leqq x$, *it is necessary and sufficient that there exist two non-negative continuous functions* $\Phi_i(x, y, y')$ $(i=1, 2)$ *as follows; namely* $\Phi_1(x, y, y')$ *and* $\Phi_2(x, y, y')$ *are defined in*