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On the Solutions of the System of Ordinary Differential Equations.[1]

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§ 1. Introduction.

In this paper we will treat the system of ordinary differential equations

$$\frac{dy_i}{dx}=f_i(x, y_1, y_2, \cdots, y_m) \quad (i=1, 2, \cdots, m). \tag{1}$$

Concerning the system, whose second members are continuous, many authors[2],[3] have investigated several fundamental theorems. Here we will extend some of them into the case of the system which has discontinuous second members.

In the following the integrals are of Lebesgue sense and y represents the vector in the space of m dimensions: namely $y=(y_1, y_2, \cdots, y_m)$, and $|y|=\sqrt{y_1^2+y_2^2+\cdots+y_m^2}$. Therefore (1) may be represented by

$$\frac{dy}{dx}=f(x, y). \tag{2}$$

And, we assume that $f(x, y)$ is defined in a domain G: $0\leqq x-x_0\leqq a$, $|y-y_0|\leqq b$, having the properties as follows:

a) $f(x, y)$ is measurable with regard to x, and continuous function of y,

b) $|f(x,y)|\leqq M(x)$, where $M(x)$ is summable, i. e. integrable in the sense of Lebesgue, for $0\leqq x-x_0\leqq a$.

For the differential equation (2) we call a curve $y=\varphi(x)$, the solution passing through the point $P(x_P, y_P)\in G$ provided

c) $\varphi(x)$ is defined in an interval I containing x_P, $\varphi(x_P)=y_P$ and $(x, \varphi(x))\in G (x\in I)$,

d) $\varphi(x)=y_P+\int_{x_P}^{x} f(x, \varphi(x))dx \quad (x\in I)$.