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On the Whitney Characteristic classes of the Normal Bundle

By

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1. It is the aim of this paper to establish a generalization of Chern's formula for the invariant of Whitney ([2], § 4.), that is, to obtain the integral formula of the Whitney characteristic class of the normal bundle. We use the following notations.

R^{n+N}; $(n+N)$-dimensional orientable Riemannian manifold of the class $\geqq 3$.

M^n; n-dimensional closed orientable submanifold of the same class imbedded in R^{n+N}.

$\boldsymbol{N}^{q-1}$; Bundle of the normal $(N-q+1)$-frame to R^{n+N} over M^n.

$\boldsymbol{N}^q$; Bundle of the normal $(N-q)$-frame to R^{n+N} over M^n.

$\boldsymbol{T}^0$; Bundle of the tangent n-frame to M^n over M^n.

$\boldsymbol{B}^0$; Bundle of the tangent $(n+N)$-frame to R^{n+N} over M^n.

The q-th Whitney characteristic class of the normal bundle is the cohomology class of the obstruction $c(F)$ where F is any cross-section to over the $(q-1)$-skeleton in the cellular decomposition of M^n, ([1], $p-190$) The bundle of coefficient of $\boldsymbol{N}^{q-1}$ is the product bundle by the orientability of R^{n+N} and M^n, and the $(q-1)$-th homotopy group of the fibre $V_{N,\,N-q+1}$. of $\boldsymbol{N}^{q-1}$ is ∞ if $q-1$ is even or $N=q$, and 2 if $q-1$ is odd and $N\neq q$. Then our class is regrarded as the ordinary cohmology class with the coefficient of integer or integer mod. 2. Now, we represent $c(F)$ by the integral formula. In the special case, $N=n=q$, our formula is Chern's one.

2. Let Δ be an oriented q-cell in the cellular decomposion of M^n, $\sum$ be its oriented boundary sphere and Δ be contained in a coordinate neighborhood. By the properties of the homotopy group of Stiefel manifold $V_{N,\,N-q}$ which is the fibre of $\boldsymbol{N}^q$ ([1], $p-132$), there exists the expension E_0 of pF over Δ where p is the projection $\boldsymbol{N}^{q-1}\rightarrow\boldsymbol{N}^q$. Now, $\boldsymbol{N}^{q-1}$ being regarded as the bundle over $\boldsymbol{N}^q$,