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On the Primary Difference of Two Frame Functions in a Riemannian Manifold.

By

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In a previous paper* we have expressed the Stiefel characteristic classes in terms of the forms Π^r and Ω^r. Now, we intend to express the *deformation cochain* of two frame functions by these forms. We make clear the geometrical meaning of the proof of theorem given in the said paper, and we show that Π^r may be regarded as a form which represents the *primary difference.*

We shall use, throughout this paper, the same notations as in the preceding paper.

1. The deformation cochain $d(f_0, h, f_1)$. Let f_0 and f_1 be two cross-sections: $K^r \to \mathfrak{B}^r$ $(1 \leqq r \leqq n-1)$, whose obstruction cocycles we denote by $c(f_0)$ and $c(f_1)$ respectively. Since $\pi_i(Y^r)=0$ for $i<r$, there exists a homotopy

$$h: f_0|K^{r-1} \simeq f_1|K^{r-1}.$$

The interval I is regarded as a cell complex consisting of one 1-cell I and the 0-cells 0 and 1. Let $\bar{0}$, $\bar{1}$ be the generators of the group of 0-cochains with integral coefficients; and let $\bar{I}$ denote a generator of the group of 1-cochains chosen so that $\delta\bar{0}=-\bar{I}$, $\delta\bar{1}=\bar{I}$. We may regard naturally $\mathfrak{B}^r \times I$ as a bundle over $K^n \times I$; and a cross-section φ of the part of $\mathfrak{B}^r \times I$ over the r-dimensional skeleton of $K^n \times I$, is constructed by

$$(1) \qquad \begin{aligned} &\varphi(x, 0)=(f_0(x), 0), \ \varphi(x, 1)=(f_1(x), 1) \quad \text{for } x \in K^r, \\ &\varphi(x, t)=(h(x, t), t) \qquad\qquad\qquad\qquad \text{for } x \in K^{r-1},\ t \in I. \end{aligned}$$

Then an obstruction cocycle $c(\varphi)$ is defined. If we set

$$(2) \qquad d(f_0, h, f_1) \times \bar{I} = (-1)^{r+1}\{c(\varphi)-c(f_0) \times \bar{0} - c(f_1) \times \bar{1}\},$$

the $(r+1)$-cochain $d(f_0, h, f_1) \times \bar{I}$ of $K^n \times I$ with coefficients in π_r is

*) *On the Stiefel characteristic classes of a Riemannian manifold*, these Memoirs, this number. We shall quote the paper as "[1]".