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On the Stiefel Characteristic Classes of a Riemannian Manifold

By

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Introduction. One of the chief problems in differential geometry in the large is the inquiry on relations between the *Stiefel characteristic classes* of a compact orientable manifold and a Riemannian metric defined on it. The determination of the formulas which express the characteristic classes in terms of differential forms has been discussed in the paper of Allendoerfer [1],* as an argument analogous to the proof of the Allendoerfer-Weil formula.

In the present paper we shall deal with this subject from a different point of view which is more geometrical. It will be shown that the consideration of a given frame function can be reduced to the simplest case in virtue of the homotopy theory of fibre bundles, and that the formulas can be found naturally from the well-known result due to Chern [4], making use of the induced metric on a submanifold. Thus, we make clear the intrinsic properties of the differential forms appearing in the formulas.

1. Preliminaries and notations. Let $\mathfrak{R}^n$ be a compact connected orientable Riemannian manifold of dimension n and class $\geqq 4$, and let $\mathfrak{B}^{n-1}$ denote the tangent sphere bundle over it. Then we can get in a certain way the associated bundle $\mathfrak{B}^q$ $(0 \leqq q \leqq n-1)$ of $\mathfrak{B}^{n-1}$ having the Stiefel manifold $Y^q = V_{n,\,n-q}$ as fibre. Each element of $\mathfrak{B}^q$ may be an $(n-q)$-frame in $\mathfrak{R}^n$,

$$\mathfrak{b}^q = P\mathfrak{e}_{q+1}\ \mathfrak{e}_{q+2}\cdots\mathfrak{e}_n,$$

where P is a point of $\mathfrak{R}^n$ and $\mathfrak{e}_{q+1}, \mathfrak{e}_{q+2}, \cdots, \mathfrak{e}_n$ are mutually orthogonal unit vectors of $\mathfrak{R}^n$ with origin P. If we define a map $p: \mathfrak{B}^q \rightarrow \mathfrak{B}^{q+1}$ by

$$p\mathfrak{b}^q = P\mathfrak{e}_{q+2}\cdots\mathfrak{e}_n,$$

*) Numbers in square brackets refer to the bibliography.