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Local imbedding of Riemann spaces

By

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C. B. Allendoerfer [13] defined the type number r of Riemann space, which is imbedded in a flat space, and proved that, if $r>3$ and there exist H^{P}_{ij} satisfying the Gauss equation, then we have H^{Q}_{Pj} satisfying the Codazzi and Ricci equations. Hence, in this case, imbedding problem in flat space reduces merely to algebraic one, that is, to solving the Gauss equation. But we were not given by him any intrinsic method to determine the type number of the space.

The second section of the present paper gives a necessary condition that a Riemann n-space be imbedded in an Euclidean $(n+p)$-space. A development of the discussion in this section leads us to the *intrinsic definition of the even type number* of a Riemann space, as will be shown in the third section.

The fourth and subsequent sections concern with the *imbedding of Riemann space in space of constant curvature.* The Riemann curvature K of an enveloping space will be determined by a system of equations of first degree with respect to K. The system of equations is obtained as a consequence of the necessary condition found in the second section. Thus we shall show that the *imbedding problem of Riemann space in space of constant curvature is generally reducible to one in flat space.*

§ 1. Preliminaries and historical notes

Let V_n be a Riemann n-space with the metric form

$$g_{ij}\,dx^i dx^j \qquad (i, j=1,\cdots, n,),$$

imbedded in a Riemann $m(>n)$-space V_m with the metric form

$$g_{\alpha\beta}\,dy^\alpha dy^\beta \qquad (\alpha, \beta=1,\cdots, m,),$$

V_n being defined by equations of the form