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On some properties of trajectories of the group-spaces

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Let C_{a_0} be a trajectory through the origin a_0^α in the group-space S of a continuous transformation group G_r. Transforming C_{a_0} by two transformations with same parameters b_0^α, one of which belongs to the second parameter-group and the other to the first, we obtain two trajectories $C_{b_0}^{(+)}$ and $C_{b_0}^{(-)}$ respectively through the same point b_0^α in S. In general they do not coincide with each other. Therefore it is a question that under what conditions they coincide: as curves or not only as curves but also point-wisely. We shall study, in this paper, these conditions and their meaning in the group theory. Nextly taking the case where they coincide with each other as cnrves, we study the condition that C_{a_0} may be considered as a closed curve. For these research we shall use the concept of connections. Namely, we treat S as the space of affine connection into which the so-called $(+)$-connection is induced.

The notations in a previous paper [1] will be used also here.

1. Let G_r be a continuous group of transformations with r parameters a^α and $\mathfrak{G}_r^{(+)}$ be the first parameter-group of G_r. Let

$$a_3^\alpha=\varphi^\alpha(a_1, a_2) \qquad (\alpha=1, \cdots, r)$$

be the equations of $\mathfrak{G}_r^{(+)}$ where a_2^α are considered as parameters. We represent, hereafter, these euations symbolically by

$$(1\cdot1) \qquad a_3=a_1\, a_2$$

The same equations, when a_1^α are considered as parameters instead of a_2^α, represent the equations of the second parameter-group $\mathfrak{G}_r^{(-)}$