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On the holonomy groups of the group-spaces.

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Let S be the group-space of a continuous group of transformations G_r, in which the $(+)$ or $(-)$-connection is induced. As we shall show in this paper, the holonomy group of S is a group of affine translations. It is a question, therefore, that how many essential parameters are there in the holonomy group. We shall reply to this question by giving a necessary and sufficient condition that the holonomy group has $p(\leqq r)$ essential parameters, and study the relations between the holonomy group and G_r.

We shall make use here principally of the notations of L. P. Eisenhart in his work " Continuous Group of Transformations [1]".

1. Let

$$x'^i = f^i(x, a) \qquad (i=1,\cdots,n) \tag{1·1}$$

be the equations of a continuous transformation group G_r with n independent variables x^i and r essential parameters a^α, and

$$a_3^\alpha = \varphi^\alpha(a_1, a_2) \qquad (\alpha=1,\cdots,r) \tag{1·2}$$

be the equations of the parameter-group of G_r. That is, the groups defined by (1·2) as a_2^α and a_1^α are considered as the parameters are called respectively the first and second parameter-groups of G_r. Let us denote them by $\mathfrak{G}_r^{(+)}$ and $\mathfrak{G}_r^{(-)}$ respectively.

Let

$$\frac{\partial x'^i}{\partial a^\alpha} = \xi_b^i(x') A_\alpha^b(a) \qquad \begin{pmatrix} i=1,\cdots,n; \\ b, \alpha=1,\cdots,r \end{pmatrix},$$

$$\frac{\partial a_3^\alpha}{\partial a_2^\beta} = A_b^\alpha(a_3) A_\beta^b(a_2) \qquad (b, \alpha, \beta=1,\cdots,r),$$

and

$$\frac{\partial a_3^\alpha}{\partial a_1^\beta} = \bar{A}_b^\alpha(a_3) \bar{A}_\beta^b(a_1) \qquad (b, \alpha, \beta=1,\cdots,r)$$

be the fundamental equations of G_r, $\mathfrak{G}_r^{(+)}$ and $\mathfrak{G}_r^{(-)}$ respectively,