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Note on the existence theorem of a periodic solution of the non-linear differential equation

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In the foregoing paper[1] we have obtained a *boundedness theorem* from which we have deduced an *existence theorem* of a periodic solution of the non-linear differential equation. Since there we have made use of the property of the ultimate boundedness, such existence theorem for a periodic solution is no more applicable to the case where the solutions are not ultimately bounded. Therefore in this paper we will search an existence theorem applicable to this new case. The principle is to establish that each solution starting from $t=0$ is bounded for $0 \leqq t < \infty$. Then since each solution is continuable and moreover there exists a bounded solution, we can obtain an existence theorem by aid of Massera's theorem.[2]

At first, as a sufficient condition for the boundedness of solutions, generalizing the problem for the general system of differential equations, we will prove the following theorem.

Theorem 1. *Consider a system of differential equations,*

$$\frac{dx_i}{dt} = f_i(t, x_1, x_2, \cdots, x_n) \quad (i=1, 2, \cdots, n), \tag{1}$$

where $f_i(t, x_1, x_2, \cdots, x_n)$ *are continuous functions of* $(t, x_1, x_2, \cdots, x_n)$ *in the domain*

$$D_1: \ 0 \leqq t < \infty, \quad -\infty < x_i < +\infty \quad (i=1,2,\cdots, n).$$

Now let R_0 *be a positive constant which may be sufficiently great and* D_2 *be the domain such as*

1) These Memoirs, Series A, Mathematics, Vol. 28, pp. 133-141.

2) Wendel; Ann. Math. Stud. no. 20 (Princeton, 1950), p. 266 or Massera; "The existence of periodic solutions of systems of differential equations", Duke Math. Journal. Vol. 17 (1950), pp. 457-475.