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On the convergence of solutions of the non-linear differential equation

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In the foregoing papar* we have researched sufficient conditions for the *ultimate boundedness* of solutions of the system of differential equations,

$$(1)\qquad \begin{aligned} \frac{dx}{dt}&=f(t,x,y) \\ \frac{dy}{dt}&=g(t,x,y), \end{aligned}$$

and we have obtained an existence theorem of a periodic solution by aid of the boundedness theorem. Namely under some conditions, it is proved that there exist two positive numbers A and B independent of particular solutions such that

$$|x(t)|<A, \qquad |y(t)|<B$$

for $t\geqq t_0$ (t_0 depending upon each particular solution), where $(x(t), y(t))$ is any solution of (1).

Let $f(t,x,y)$ and $g(t,x,y)$ be two *continuous functions* of (t,x,y) in the domain

$$\Delta_1:\ 0\leqq t<\infty, \quad -\infty<x<+\infty, \quad -\infty<y<+\infty.$$

Now we will show that under some conditions every solution of (1) converges to the periodic solution as $t\to\infty$ provided the solutions of (1) are ultimately bounded. At first, we shall prove two following lemmas.

Lemma 1. *Let Δ_2 be the 5-dimensional domain of (t,x,u,y,v) such as*

$$t_0\leqq t<\infty,\ |x|\leqq A,\ |u|\leqq A,\ |y|\leqq B,\ |v|\leqq B,$$

where t_0 may be arbitrarily great, but it is a constant. Now suppose

* These Memoirs, Series A, Mathematics, Vol. 28, pp. 133–141.