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On the non-linear differential equation

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1. About special forms of non-linear differential equations of the second order, the boundedness of solutions and the existence of a periodic solution have been discussed by various authors; Cartwright, Littlewood, Reuter and others.

Now generalizing the problems, we consider a system of differential equations,

$$(1) \qquad \begin{cases} \dfrac{dx}{dt}=f(t, x, y) \\ \dfrac{dy}{dt}=g(t, x, y) \end{cases} ,$$

where $f(t, x, y)$ and $g(t, x, y)$ are *continuous* in the domain

$$\Delta_1: \quad 0 \leqq t < +\infty, \quad -\infty < x < +\infty, \quad -\infty < y < +\infty.$$

The non-linear differential equation of the second order is a special case of (1).

At first, we shall prove two lemmas in order to discuss the boundedness theorem for the solutions of (1).

Lemma 1. *Let A_1 and B_1 be two positive constants (A_1 and B_1 may be arbitrarily great) and $\mathfrak{A}$ be the domain*

$$|x| < A_1, \quad |y| < B_1.$$

Suppose that there exists a continuous function $\Phi(x, y)$ satisfying the following conditions in the domain

$$\Delta_2: \quad 0 \leqq t < +\infty, \quad (x, y) \in \mathfrak{A}^c,$$

where $\mathfrak{A}^c$ is the complement of $\mathfrak{A}$ in $[-\infty < x < +\infty, -\infty < y < +\infty]$; namely the conditions are that

1° $\Phi(x, y) > 0$,

2° *$\Phi(x, y)$ tends to zero uniformly for y and x respectively when $|x|$ or $|y|$ becomes infinity,*