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On the differential equation of Carathéodory's type

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In this paper we shall study the differential equation of Carathéodory's type:

$$\frac{dy}{dx}=f(x, y)$$

where $f(x, y)$ is defined in the strip

$$S: \quad a \leqq x \leqq b, \quad -\infty < y < +\infty,$$

measurable with respect to x, continuous with respect to y and dominated in S by a summable function $k(x)$ of x alone. It is known[1] that the above equation has solutions in the interval

$$I: \quad a \leqq x \leqq b,$$

in the following sense: there exist in I absolutely continuous functions $\varphi(x)$ such that $\varphi(x)=\varphi(a)+\int_a^x f(t,\varphi(t))dt$ in I, and then $\varphi'(x)=f(x, \varphi(x))$ in a measurable subset of I, having the same measure as I and depending upon the particular solution $\varphi(x)$.

Recently Prof. G. Scorza Dragoni[2] has proved that we can determine a measurable subset E of I such that E is of the same measure as I and every absolutely continuous solution satisfies the differential equation on the set E.

The purpose of this paper is to give a simple proof of this theorem.

§ 1. Let $g(x)$ be a measurable function of x in I such that $|g(x)| \leqq h(x)$, $h(x)$ being summable in I. Moreover, suppose that $g(x)$ is continuous on each of sets $e_1, e_2, \cdots\cdots$, closed and satisfying $\lim_{n\to\infty} m(e_n)=b-a$. Let e'_n denote the set of the points of e_n of density 1 for e_n and e the union of all $e'_n (n=1, 2, \cdots\cdots)$. Then e is a mea-