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Notes on Chow points of algebraic varieties.

By

Yoshikazu NAKAI

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Let V be an algebraic variety embedded in a projective space. Then as is well known we can represent V by a point in a suitable projective space by the method of associated forms[1]. Henceforth we shall call it briefly the Chow point of V and denote it by $c(V)$. In this short note we shall prove two theorems, one concerning the Chow point of a variety, and the other concerning the Chow point of the divisors on a variety.

THEOROM 1. Let V be a variety embedded in a projective space and κ the prime field of characteristic p. Let M_λ $(\lambda=1, 2,\cdots)$ be a sequence of independent generic points of V over some field of definition k for V, then for sufficiently large α we have $c(V) \subset \kappa(M_1,\cdots,M_\alpha)$.

PROOF. As is well known a projective model has the smallest field of definition $k_0=\kappa(c(V))$.[2] Let $\mathfrak{P}$ be the defining ideal of V in $k[X]$. Then we can select special basis $(P_1(X),\cdots,P_s(X))$ for $\mathfrak{P}$ having the following properties.

(1) k_0 is get by the adjunction of the coefficients of $P_j(X)$ to κ.

(2) Let $\mathfrak{M}_\lambda(X)$ be monomials in X with suitable ordering and J_i be the set of indices such that $P_i(X)$ is exactly the linear forms in $\mathfrak{M}_{\lambda_i}(X)$ with $\lambda_i \in J_i$. Then for any proper subset J_i' of J_i, the linear forms $\sum u_\beta \mathfrak{M}_\beta(X)$ with $\beta \in J_i'$ and $u_\beta \in k_0$ can not be contained in $\mathfrak{P}$. Such basis can be get by the procedure given in W-I,[3] lemma 2. Let

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1) Cf. B. L. van der Waerden, "Einführung in die algebraische Geometrie". Julius Springer in Berlin, 1939.

2) Cf. S. Nakano, "Note on gruop varietirs", Mem. Coll. Sci., Univ. of Kyoto, vol. XXVII, 1942.

3) This means the lemma 2 of Chap. I of "Foundations of algebraic geometry" written by A. Weil,