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Note on integral closures of Noetherian domains

By

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Previously Prof. Akizuki[1] proved that if $\mathfrak{o}$ is a Noetherian local integrity domain[2] of dimension 1 and if $\hat{\mathfrak{o}}$ is its integral closure[3], then any ring $\mathfrak{s}$ such that $\mathfrak{o}\subseteq\mathfrak{s}\subseteq\hat{\mathfrak{o}}$ is Notherian[4].

As for the case of higher dimension, there arise the following problems:

Let $\mathfrak{o}$ be a Noetherian local integrity domain of dimension n and let $\hat{\mathfrak{o}}$ be its integral closure. Then

Problem I. Does it holds in general that any ring $\mathfrak{s}$ such that $\mathfrak{o}\subseteq\mathfrak{s}\subseteq\hat{\mathfrak{o}}$ is Noetherian?

Problem II. Does it holds in general that $\hat{\mathfrak{o}}$ is Noetherian?

In the present note, we show a counter example against the problem I when $n=2$ in § 2 and then a counter example against the problem II when $n=3$ in § 3[5].

§ 1. A preliminary.

Let $\mathfrak{k}_0$ be a perfect field of characteristic p $(\neq 0)$ and let $u_1\cdots,$ $u_n,\cdots$ (infinitely many) be algebraically independent elements over $\mathfrak{k}_0$. Set $\mathfrak{k}=\mathfrak{k}_0(u_1,\cdots, u_n,\cdots)$. Further let $x_1,\cdots, x_n$ be indeterminates and denote by $\mathfrak{o}_n$ and $\mathfrak{r}_n$ the rings $\mathfrak{k}^p\{x_1,\cdots, x_n\}[\mathfrak{k}]$ and $\mathfrak{k}\{x_1,\cdots, x_n\}$[6] respectively.

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1) Y. Akizuki, Einige Bemerkunge über primäre Integritätsbereiche mit Teilerkettensatz, Proc. Phys.-Math. Soc. Japan, 3rd Ser., 17 (1935), pp. 327-336.

2) We say in the present note that a ring $\mathfrak{o}$ is a local ring if it has only one maximal ideal $\mathfrak{m}$ and if the intersection of all powers of $\mathfrak{m}$ is zero, where we consider the $\mathfrak{m}$-adic topology for $\mathfrak{o}$.

3) This means the integral closure in its quotient field.

4) This result shows also the similar result for "einartig" Noetherian integrity domains.

5) It was communicated to the writer that this problem II was proved affirmatively by Mr. Mori, when $n=2$.

6) $\mathfrak{k}\{x_1,\cdots, x_n\}$ denotes the ring of formal power series in $x_1,\cdots x_n$ with coefficients in $\mathfrak{k}$.