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Some remarks on local rings, II

By

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This paper contains some applications of the theory of Henselian rings to the theory of local rings. Main purpose of the present paper is to prove the following two assertions:

I) If an integrally closed local integrity domain $\mathfrak{o}$ is of finitely generated type over a valuation ring of characteristic 0 or over a field of arbitrary characteristic[1], then the completion of $\mathfrak{o}$ is an integrally closed integrity domain.

II) If an integrally closed local integrity domain $\mathfrak{o}$ is of finite type over a regular local ring $\mathfrak{r}$[1] then the completion of $\mathfrak{o}$ is an integrity domain, provided that $\mathfrak{r}$ contains a complete Noetherian local ring $\mathfrak{s}$ such that $\mathfrak{r}$ is a quotient ring of $\mathfrak{s}$ with respect to a prime ideal.

These results generalize and supplement a result (Theorem 5) in my previous paper[2]. By the way, we add a proof of the following:

Let C be the complex number field and let $\mathfrak{o}_n$ and $\bar{\mathfrak{o}}_n$ be the rings of convergent and formal power series respectively in n variables $z_1, \cdots, z_n$ over C. Then if $\mathfrak{p}$ is a prime ideal of $\mathfrak{o}_n$ then $\mathfrak{p}\bar{\mathfrak{o}}_n$ is also a prime ideal.

As for terminology, see my other papers on Henselian rings[3].

§ 1. Henselian regular local rings.

THEOREM 1. *Let* $\mathfrak{r}$ *be a Henselian regular local ring and let* $\mathfrak{m}$ *be*

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1) As for definition, see § 3 below.

2) Some remarks on local rings, to appear in Nagoya Math. J., 6, which will be referred as [L.R.] in the present paper.

3) On the theory of Henselian rings, Nagoya Math. J., 5, pp. 45–57: On the theory of Henselian rings II, to appear in Nagoya Math. J., They will be referred as [H.R. I] and [H.R. II] respectively in the present paper,