MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXVIII, Mathematics No. 2, 1953

On differential algebra of arbitrary characteristic

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(Received October 1, 1953)

A number of results have been established for differential algebra,[1] and so it comes clear in the case of characteristic zero, but the case of nonzero characteristic was investigated only in a few papers. Among them, Kolchin (2) considered the basis theorem for systems of ordinary or partial differential polynomials over a differential ring, and Seidenberg (8) considered some basic theorems in the *ordinary* differential situation. In the following we shall generalize the results of Seidenberg in the *partial* differential situation. Our basis theorem is a generalization of the Kolchin's result if it is restricted to differential polynomials over a differential *field*. This restricted case seems useful in the application.

1. Differential polynomials. By a *differential ring* is meant a ring $\mathfrak{R}$ with a finite number m of given differentiations $\delta_1, \cdots, \delta_m$ which are mutually commutative. According as $m=1$ or >1, we shall say that $\mathfrak{R}$ is an *ordinary* or a *partial* differential ring. The differential ideal which is generated by a subset α of $\mathfrak{R}$ will be denoted by $[\alpha]$. The semiprime (or perfect) differential ideal generated by α will be denoted by $\{\alpha\}$. By a *differential field* is meant a differential ring which is a field; its characteristic p may be arbitrary.

Let $\mathfrak{F}$ be a differential field and $X_1, \cdots, X_n$ a finite number n of independent differential indeterminates over $\mathfrak{F}$. The differential ring of all differential polynomials over $\mathfrak{F}$ will be denoted by $\mathfrak{R}=\mathfrak{F}\{X_1, \cdots, X_n\}$. Dealing with differential polynomials, it is useful to introduce a linear order relation $\lesssim$ for differential polynomials (cf. Ritt (7) and Kolchin (2)). We define it below after some preliminary definitions.

1) A complete bibliography up to 1948 may be found in Ritt (7). To it may be added subsequent literatures: Herz (1), Okugawa (3-5) and Seidenberg (8); and Kolchin's works (especially, the one which is quoted in the footnote of p. 105).