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On some properties of the non-linear differential equations of the "Parametric excitation"

By

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To my Teacher, Toshizô MATSUMOTO
on the occasion of his 63rd birthday

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Introduction. The author and his collaborator S. Mizohata[1] have obtained a proof for the existence of the periodic solutions of the non-linear differential equations of the following type under comparatively weak conditions,

$$\ddot{x}+f(x)\dot{x}+g(x)=p(t).$$

This problem has been raised from the researches of the non-linear vibrations in the field of engineerings. In this report, we shall discuss the wider class of problem which contains the so-called "Parametric Excitation"[2] which has not yet been rigorously discussed. For example, one case is expressed by the equation,[3]

$$\ddot{x}+\beta_0\dot{x}+(p_0^2+\alpha_0\cos 2\omega t)x+\gamma_0 x^3=p_0\cos(\omega t+\varphi)$$

on which we shall have the following conclusion in this report: This equation has at least one periodic solution having such property that

$$-x(t)=x\left(t+\frac{\pi}{\omega}\right) \quad \text{as} \quad \beta_0>0, \quad \gamma_0>0.$$

We shall describe the obtained results as two theorems I, II and add several examples.

Now we shall consider the following differential equation

$$(1) \qquad \ddot{x}+f(x)\dot{x}+g(x,t)=p(t),$$

where the functions $f(x)$, $g(x, t)$ satisfy the Lipschitz condition[4] with respect to x, and $g(x, t)$ has a continuous partial derivative $g_t(x, t)$ and $g(x, t)$, $p(t)$ are continuous periodic functions of t having the period ω, and $p(t)$ satisfies the condition $\int_0^{\omega} p(t)\,dt=0$. We put the