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On transformations of differential equations

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In the first two sections we consider the system of ordinary differential equations

$$\frac{dy_i}{dx}=f_i(x, y_1, y_2, \cdots, y_n) \qquad (i=1, 2, \cdots, n) \tag{1}$$

where $f_i(x, y_1, y_2, \cdots, y_n)$ are defined and continuous in a region

$$E_{n+1}\ :\ 0\leqq x\leqq a\,,\quad |y_i|<+\infty \qquad (i=1, 2, \cdots, n).$$

Let us consider $(y_1, y_2, \cdots, y_n)$ as a vector y, then $(f_1, f_2, \cdots, f_n)$ defines a vector-function of (x, y), conveniently written $f(x, y)$. Thus (1) assumes the simple form

$$\frac{dy}{dx}=f(x, y). \tag{2}$$

In § 3, the differential equation of the second order is investigated as a special case of (1).

§ 1. Transformations of (1)

Let $f(t)$ be the greatest value of 1, t and $\max\limits_{\substack{0\leqq x\leqq a\\ |y|\leqq t}}|f(x, y)|$, where $|y|=\sqrt{y_1^2+y_2^2+\cdots+y_n^2}$ and $|f|=\sqrt{f_1^2+f_2^2+\cdots+f_n^2}$, then $f(t)$ is a positive continuous function of t, not less than unity, in $0\leqq t<+\infty$. Now for a given positive constant σ, consider the function $\lambda(r)$ defined by the relation

$$\frac{1}{\{\lambda(r)\}^\sigma}=\int_r^{r+1}\frac{dt}{\{f(t)\}^2},$$

then $\lambda(r)$ is a continuous function of r in $0\leqq r<+\infty$, $\lambda(r)\geqq 1$ in $0\leqq r<+\infty$ and $\lim\limits_{r\to+\infty}\lambda(r)=+\infty$. And evidently $\lambda(r)$ has the continuous derivative