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Kelvin principle and some inequalities in the theory of functions I

By

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1. Introduction. Recently Z. Nehari[8] has, by means of Dirichlet principle, obtained various inequalities of function theory and potential theory which may be reduced to statements regarding the properties of harmonic functions with constant boundary values, that is, functions obtainable from the Green's function. While his method is very useful to deduce some inequalities important in the theory of conformal maps, it seems difficult to derive, by this method, several inequalities which may be reduced to statements regarding the properties of harmonic functions with a vanishing normal derivative on some of boundary components of a given domain.

It is the aim of the present paper to show that the inequalities of this type can be deduced from the classical Kelvin principle[7,10]. Although some of results obtained in this paper are not new, the method used there will suggest a more or less systematic treatment of the inequalities of this type.

2. Kelvin principle and a monotonic functional. Let q be any vector function defined in a given domain D, satisfying the following conditions;

$$(1) \qquad \begin{aligned} \operatorname{div} q &= 0 \quad \text{in } D \\ qn &= f(s) \quad \text{on } C \text{ (boundary of } D), \end{aligned}$$

n being the unit vector in the direction of outward normal and $f(s)$ a function of arc-length s defined on C satisfying the condition $\int_C f(s)ds=0$. Under the latter condition there exists a harmonic function ϕ in D, satisfying the condition

$$\frac{\partial \phi}{\partial n}=f(s) \quad \text{on } C,$$