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Automorphism-groups of differential fields and group-varieties

By

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Recently E. R. Kolchin [1] has developed a beautiful Galois theory of differential fields. The Galois group is equipped with an algebraico-geometric structure. His main theorem asserts Galois correspondence between the intermediate differential fields and the algebraic subgroups (i. e. the closed subgroups in Zariski topology) of the automorphism group. His results show also that the groups of strong isomorphisms in his sense have, if irreducible, almost all properties of group variety in the sense of A. Weil. Combining Kolchin's results with Weil's method of construction of group varieties ([4]), and using an idea of Nakano [5], we shall show in this note that Kolchin's irreducible groups are, as he conjectured, group varieties in Weil's sense. Further we shall add some remarks on the specialization and the solvability in the whole (eventually reducible) group.

1. Let g/F be a strongly normal extension (cf. [1]). Thus F is of characteristic O, and g is finitely generated over F (not only in differential sense, but also in algebraic sense). g and F have the same constant field C, which is assumed to be algebraically closed. Let G^* and G be the group of the strong isomorphisms and the automorphism group, respectively. For the present we shall assume that G^* is irreducible. All the fields to be considered are contained in a universal extension g^* with constant field C^*. g and C^* are linearly disjoint over C, and so specializations over C and those over g are equivalent for constants.

2. Let $\bar{\sigma}$ be a generic element of G^*. $C_{\bar{\sigma}}$ is finitely generated over C, and we can set $C_{\bar{\sigma}}=C(\gamma_{\bar{\sigma}})$, $(\gamma_{\bar{\sigma}})=(\gamma_{\bar{\sigma}1}, \cdots, \gamma_{\bar{\sigma}m})$. Moreover we can, and shall, take $(\gamma_{\bar{\sigma}})$ so that the affine variety defined over C with generic point $(\gamma_{\bar{\sigma}})$ may be (everywhere) normal. Let $g=$