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Note on complete local integrity domains

By

Masayoshi NAGATA

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Previously some interesting results concerning prime ideals in rings of formal power series were proved by C. Chevalley [1]. In the present paper, we want to offer a new treatment on the similar assertions. We see on the way a new result that when $\mathfrak{o}$ is a complete (Noetherian) local integrity domain with a basic field k, $\mathfrak{o}$ is separably generated[1] over k if and only if there exists a system of parameters $x_1, \cdots, x_n$ of $\mathfrak{o}$ such that $\mathfrak{o}$ is separable over the ring $k\{x_1, \cdots, x_n\}$ (formal power series).

Throughout the present paper, a local ring means a Noetherian local ring which contains a field.

§1. Kroneckerian products.

Let $\mathfrak{o}_1$ and $\mathfrak{o}_2$ be complete local rings with basic fields k_1 and k_2 respectively. If K is a field containing both k_1 and k_2, we can define the Kroneckerian product of (k_1-algebra) $\mathfrak{o}_1$ and (k_2-algebra) $\mathfrak{o}_2$ over K, as was defined by C. Chevalley [2]. We denote this Kroneckerian product by $\mathfrak{o}_1/k_1 \times_K \mathfrak{o}_2/k_2$[2]. (For the detail, see Chevalley [2]). When $k_1 = k_2 = K$, we denote this by $\mathfrak{o}_1 \times_K \mathfrak{o}_2$.

We define further Kroneckerian products of complete local rings with discrete rings:

Let $\mathfrak{o}_1$ be a complete local ring with basic field k_1 and let $\mathfrak{o}_2$ be a discrete ring[3] which contains a field k_2. Assume that K is a field which contain both k_1 and k_2. We define the Kroneckerian product of k_1-algebra $\mathfrak{o}_1$ and discrete k_2-algebra $\mathfrak{o}_2$ over K as follows:

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1) For the definition, see Chevalley [1] or §2 in the present paper.

2) Though Chevalley [2] denotes this ring by $\mathfrak{o}_1 \times_K \mathfrak{o}_2$, we dare use a more complicated notation because the product depends on the choice of basic fields.

3) $\mathfrak{o}_2$ may be a topological ring which is not discrete; we only regard it as an abstract ring (or a discrete topological ring).