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On the existence of a curve connecting given points on an abstract variety

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In the course of study in algebraic geometry, we are frequently encountered to treat the following problem. Let V be an abstract variety, and P, Q be two points on V, then does there exist an irreducible curve connecting these two points? It may seem to be almost self-evident, but it seems to us that there is no any proof in the literature. In this note we shall answer the above in the following generalized form.

THEOREM. Let V^n be an abstract variety, and $U_i^{s_i}(i=1, \cdots, m)$ be finite number of subvarieties of dimensions s_i respectively, such that $s=\max(s_i)<n-1$. Then there exists an irreducible subvariety of V containing all U_i, of any dimension r such that $s+1\leqq r\leqq n-1$. Moreover there exists such one which is algebraic over any common field of definition for V and U_i $(i=1, \cdots, m)$.

First we shall prove the theorem in the case when V is a projective model, and then go into the general case.

LEMMA 1. Let V^n be a projective model, and $P_i(i=1, \cdots, m)$ be arbitrary points on V. Then there exists an irreducible subvariety of V, containing all P_i, of any dimension r such that $1\leqq r\leqq n-1$. Moreover let k be a field of definition for V, then there exists such one which is algebraic over $k(P_1, \cdots, P_m)$.

PROOF. It is sufficient to treat the case $r=n-1$. First we shall assume that V is normal. Let t be an integer satisfying the following condition. Let Q be an arbitrary point of V, different from any of P_i, there exists a hypersurface of order $t-1$, containing all P_i, but not Q. Such integer surely exists, e.g., $t=m+1$. Put $\mathfrak{A}=\sum P_i$ then the linear system $\sum_{\mathfrak{A}}$ which consists of the intersections of V with all hypersurfaces of order t containing all points in $\mathfrak{A}$, will be shown to be noncomposite with the pencils. In fact,