MEMOIRS OF THE CLLOEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXVIII, Mathematics No. 3, 1953.

On the characteristic classes of a submanifold

By

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(Received April 10, 1954)

In this paper, we first give some remarks on the differential forms introduced in our previous papers[1][2], and we show secondly that the forms can also represent the characteristic cohomology classes of tangent and normal bundles over a submanifold imbedded in a Riemannian manifold $\boldsymbol{R}^n$, by integrating over suitable chains in the tangent frame bundles over $\boldsymbol{R}^n$.

§ 1. The formulas of obstruction cocycles and deformation cochains

Consider a Riemannian manifold $\boldsymbol{R}^n$ of dimension n which we shall suppose, as in previous papers, to be compact connected orientable and of class $\geqq 4$. The group of the tangent sphere bundle $\mathfrak{B}^{n-1}$ over $\boldsymbol{R}^n$ may be the proper orthogonal group, and so any element of the associated principal bundle $\mathfrak{B}^0$ of $\mathfrak{B}^{n-1}$ can be expressed by an n-frame $\boldsymbol{P}\boldsymbol{e}_1\boldsymbol{e}_2\cdots\boldsymbol{e}_n$ which determines one of the orientations of $\boldsymbol{R}^n$.

Take an even permutation σ of n figures $(1, 2, \cdots, n)$ and set $\sigma(A)=A'$ $(A=1, 2, \cdots, n)$. Let us denote any element of the tangent $(n-q)$-frame bundle $\mathfrak{B}^q$ associated with $\mathfrak{B}^{n-1}$ by

$$\boldsymbol{P}\boldsymbol{e}_{(q+1)'}\boldsymbol{e}_{(q+2)'}\cdots\boldsymbol{e}_{n'}\in\mathfrak{B}^q.$$

And the natural projection $\mu:\mathfrak{B}^q\rightarrow\mathfrak{B}^{q+1}$ is defined by

$$\mu\boldsymbol{P}\boldsymbol{e}_{(q+1)'}\boldsymbol{e}_{(q+2)'}\cdots\boldsymbol{e}_{n'}=\boldsymbol{P}\boldsymbol{e}_{(q+2)'}\cdots\boldsymbol{e}_{n'}\in\mathfrak{B}^{q+1}.$$

Then, for any cross-section F into the bundle $\mathfrak{B}^{r-1}$ defined on the

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1) S. Takizawa: *On the Stiefel characteristic classes of a Riemannian manifold,* these Memoirs, Vol. **28**, No. 1 (1953).

2) S. Takizawa: *On the primary difference of two frame functions in a Riemannian manifold,* Ibid,