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Stationary random distributions

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In the same way as the concept of distributions by L. Schwartz [11][1] was introduced as an extended one of functions, we may define stationary random distributions as an extension of stationary random functions viz. stationary processes. Such consideration will enable us to establish a unified theory of stationary processes, Brownian motion processes, processes with stationary increments and other similar ones, as is shown in this paper. We shall first introduce some fundamental notions in § 1. In § 2 we shall define the covariance distribution of stationary random distributions, which corresponds to Khintchine's covariance function [7]. In § 3 and § 4 we shall prove the spectral decomposition theorems of covariance distributions and stationary random distributions respectively. In § 5 we shall discuss the derivatives of stationary distributions. In § 6 we shall show that any stationary distribution is identified with a k-th derivative (in the sense of distributions) of a certain continuous process with stationary k-th order increments for some k.

§ 1. Fundamental Notions and Notations

In this paper we shall restrict ourselves to complex-valued random variables with mean 0 and finite variance. The totality of such variables constitute a *Hilbert space* $\mathfrak{H}$ with the following definition of inner product:

$$(1\cdot1) \qquad (X, Y)=\mathfrak{E}(X\cdot\overline{Y}); \quad \mathfrak{E}: \text{ expectation.}$$

We shall here consider only the strong topology on $\mathfrak{H}$. A continuous random process $X(t)$, $-\infty<t<\infty$, is an $\mathfrak{H}$-valued continuous

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1) The numbers in [ ] refer to those of the Bibliography at the end of this paper.