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Basic theorems on general commutative rings

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We want to group up some basic theorems in the general theory of commutative rings in the present note. Though most of results contained in the present note are not new, many of them will be sharpened, some of them will have simpler proofs and some of them will have more elementary proofs, than those which are already known.

Though the notions of local rings and of valuation rings are also basic for the theory of commutative rings, we will not observe them. Further, the normalization theorem (for finitely generated rings) is a basic theorem. But we will not discuss it. We concern mainly with the notion of rings of quotients, properties of integral dependence and the notion of rank of ideals.

In §1, we observe the notion of prime ideals. In §2, we study the notion of rings of quotients. In §3, we define the notion of prime divisors of ideals. In §§4–5, we study some properties of integral dependence. In §6 we observe some properties of J-radicals of rings. In §8, we study the notion of rank of ideals. In §9, we observe some properties of normal Noetherian rings.

Terminology. If $\mathfrak{o}$ is a ring without identity, then we can imbed $\mathfrak{o}$ in a ring $\mathfrak{o}'$ which has identity so that every ideal of $\mathfrak{o}$ is an ideal of $\mathfrak{o}'$. Therefore the existence of the identity does not play essential rôle in general theory of rings, except for some extreme cases. Therefore we will assume the existence of the identity in any ring of consideration, unless the contrary is explicitly stated. Since we want to treat commutative rings, we assume also commutativity. A subring of a ring is assumed to have the same identity.

Results assumed to be well known. Besides some elementary notions and results on rings and fields, we need to know elementary properties of ideals and of Noetherian rings, which are con-