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On cyclic points of group-spaces

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In the group-space S of a continuous group G_r of transformations, we have considered two kinds of repères $\mathfrak{R}(a)$ and $\bar{\mathfrak{R}}(a)$ at every point a in one of the previous papers [1]. Though they can be chosen so that they coincide at the origin a_0, they do not coincide at other points in general. Moreover, let C_a be a path through a point a in S. Through another point b there exist two paths $C_b^{(+)}$ and $C_b^{(-)}$ which are $(+)$-parallel and $(-)$-parallel to C_a respectively. These paths are also not coincident in general except the case when G_r is abelian. In this paper we define that a point b is a cyclic point with respect to a point a when $C_b^{(+)}$ and $C_b^{(-)}$ are coincident with one another not only as curves but also pointwisely. We shall show that a necessary and sufficient condition for the existence of such cyclic points is that the relative position between the two kinds of repères at a point is coincident with the one at another certain point. The notations in the previous papers ([1], [2]) will be used also here.

1. In the group-space S of a continuous group G_r of transformations with r parameters a^α's, let a be a point whose coordinates are $(a^1, \cdots, a^r)$ and T_a a transformation of G_r with parameters $a^\alpha(a^1, \cdots, a^r)$. We denote by a_0 the point which represents the identical transformation G_r. When a point a is transformed to b by the transformation of the first parameter-group $\mathfrak{G}_r^{(+)}$ of G_r with certain constant parameters $p^\alpha(p^1, \cdots, p^r)$, we have

$$(1\cdot1) \qquad T_b = T_p T_a$$

Similarly, if b' is a point transformed from a' by the same transformation, we have

$$(1\cdot2) \qquad T_{b'} = T_p T_{a'}$$